

These equations deal with decibels of loss, rather than of gain.
 These equations are from:

Don Lancaster, *Lancaster's Active Filter Cookbook, 2nd edition*
 (Oxford: Newnes, 1996).

Definitions

$$f = \text{frequency} \quad \omega = \frac{f_{\text{measured}}}{f_{\text{resonant}}} \quad S = \omega \sqrt{-1}$$

$$d = \text{damping} \quad Q = \frac{1}{d} \quad K = \text{gain}$$

$$\phi = \text{phase} \quad \text{decibels} = 20 \log_{10} \left[\frac{e_{\text{out}}}{e_{\text{in}}} \right]$$

Second order high-pass

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{KS^2}{S^2 + dS + 1} = \sqrt{\left(\frac{1}{\omega^4}\right) + \frac{d^2 - 2}{\omega^2} + 1}$$

$$\phi = \tan^{-1} \frac{d \div \omega}{1 - \frac{1}{\omega^2}}$$

Fourth order high-pass

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{KS^2}{S^2 + dS + 1} \times \frac{KS^2}{S^2 + dS + 1}$$

$$\phi = \left[\tan^{-1} \frac{d \div \omega}{1 - \frac{1}{\omega^2}} \right] + \left[\tan^{-1} \frac{d \div \omega}{1 - \frac{1}{\omega^2}} \right]$$

Second order band-pass

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{KS}{S^2 + dS + 1} = \sqrt{1 + Q^2 \left[\frac{\omega^2 - 1}{\omega} \right]^2}$$

$$\phi = - \tan^{-1} Q^2 \left[\frac{\omega^2 - 1}{\omega} \right]$$

Second order low-pass

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{K}{S^2 + dS + 1} = \sqrt{\omega^4 + (d^2 - 2)\omega^2 + 1}$$

$$\phi = - \tan^{-1} \frac{d\omega}{1 - \omega^2}$$

Fourth order low-pass

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \frac{K}{S^2 + dS + 1} \times \frac{K}{S^2 + dS + 1}$$

$$\phi = \left[- \tan^{-1} \frac{d\omega}{1 - \omega^2} \right] + \left[- \tan^{-1} \frac{d\omega}{1 - \omega^2} \right]$$